On the dynamical effects in the characteristics of transition radiation produced by a relativistic electron in a single crystal plate

S. Blazhevich *, A. Noskov

Belgorod State University, Laboratory of Radiation Physics, 14 Studencheskaya Str., 308007 Belgorod, Russian Federation

Abstract

This article reports on the transition radiation (TR) of a relativistic electron crossing a crystalline plate, which in the Bragg's scattering geometry is considered within the framework of the X-ray diffraction dynamical theory. The expressions for TR and diffracted transition radiation (DTR) spectral–angular distribution received allow to discuss the manifestation of the dynamical approach. It is shown, that the spectral–angular characteristics of TR and DTR essentially depend on the orientation of the crystal plate surface when the angle between the electron velocity and the atomic planes is kept constant. This phenomenon is explained by the modification of the frequency range of total external reflection of the radiation under a changing target surface orientation.

Keywords: Relativistic electron; Transition radiation; Single crystal; Angle distribution

1. Introduction

When crossing the boundary between two different mediums a fast charged particle induces transition radiation [1]. If the particle crosses a plate, the interference between waves radiated on in- and out-surfaces of the plate appears. The TR waves, produced on input surface target, are scattered by a system of atomic planes in the crystal (see [2–5]) which bring about a change in the conditions of interference of these waves [6].

In the given work a detailed theoretical analysis of the TR waves in Bragg's scattering geometry is carried out. The spectral–angular distribution for TR photon is achieved by applying the dynamical diffraction theory [7]. It shows, that the interference of the waves from the in- and out-surfaces in Bragg's scattering geometry essentially depends on the orientation of the crystal plate surface under a fixed angle between the electron velocity and the crystallographic plane. In other words it means that the characteristics of the TR of the relativistic electron in a single crystal plate essentially depend on the cut angle of the plate surfaces. In the current work this effect is explained as the result of changing the anomalous dispersion frequency range (range of total reflection). This effect is analogous to the known effect for free x-waves in a crystal [7]. It can be explained as a result of changing the total reflection frequency range, in which the vectors, perpendicular to the entrance surface in the space of the reciprocal lattice, do not cross the disperse surface, that corresponds to the exponential damping of waves, passing in the crystal in a forward direction. In this frequency range the wave vectors take on a complex value, which correspond to the exponential weakening of the wave intensity on passing through the crystal.

In the frame of the proposed model the conditions of the TR interference peak reduction are shown in this work, which allows to reveal the optimum conditions for the experimental observation of the parametric X-ray radiation at a small angle to the radiating electron velocity.

* Corresponding author.
E-mail address: blazh@bsu.edu.ru (S. Blazhevich).

On the other hand, the conditions of the interference peak increasing and narrowing of the TR spectral width are shown, too.

2. The general expressions for spectral–angular distribution TR in dynamic approximation

Let us consider the radiation of the fast charged particle moving with constant velocity $\boldsymbol{V}$ through a crystal of L thick (Fig. 1). Let us consider the equations for the Fourier direct image of electromagnetic field. While solving the problem, let us take up the equations for the Fourier direct image of the electromagnetic field $\boldsymbol{E}(\boldsymbol{k},\omega) = \int \mathrm{d}t\,\mathrm{d}^3r\boldsymbol{E}(\boldsymbol{r},t)\exp(\mathrm{i}\omega t - \mathrm{i}\boldsymbol{kr})$.

Since the Coulomb field of an ultrarelativistic particle can be allowed to transverse with a good degree of accuracy, the incident $\boldsymbol{E}_0(\boldsymbol{k},\omega)$ and diffracted $\boldsymbol{E}_1(\boldsymbol{k},\omega)$ electromagnetic waves are defined by two amplitudes with different values of transverse polarization.

$$\begin{aligned}\boldsymbol{E}_0(k,\omega) &= E_0^{(1)}(\boldsymbol{k},\omega)\boldsymbol{e}_0^{(1)} + E_0^{(2)}(\boldsymbol{k},\omega)\boldsymbol{e}_0^{(2)},\\ \boldsymbol{E}_1(k,\omega) &= E_1^{(1)}(\boldsymbol{k},\omega)\boldsymbol{e}_1^{(1)} + E_1^{(2)}(\boldsymbol{k},\omega)\boldsymbol{e}_1^{(2)}.\end{aligned} \tag{1}$$

The unit vectors $\boldsymbol{e}_0^{(1)}, \boldsymbol{e}_0^{(2)}, \boldsymbol{e}_1^{(1)}$ and $\boldsymbol{e}_1^{(2)}$ are chosen in the following way. Vectors $\boldsymbol{e}_0^{(1)}$ and $\boldsymbol{e}_0^{(2)}$ are perpendicular to vector $\boldsymbol{k}$, and vectors $\boldsymbol{e}_1^{(1)}$ and $\boldsymbol{e}_1^{(2)}$ are perpendicular to vector $\boldsymbol{k}+\boldsymbol{g}$. Vectors $\boldsymbol{e}_0^{(2)}$ and $\boldsymbol{e}_1^{(2)}$ are situated on the plane of vectors $\boldsymbol{k}$ and $\boldsymbol{k}+\boldsymbol{g}$ (π-polarization) and $\boldsymbol{e}_0^{(1)}$, $\boldsymbol{e}_1^{(1)}$ are perpendicular to this plane (σ-polarization); $\boldsymbol{g}$ is vector of the reciprocal lattice, defining a set of reflecting atomic planes. Using the two-wave approximation of the dynamic theory of diffraction, we will apply the well-known system of equations for the Fourier transform images of intensity of incident and diffracted waves [8]:

$$\begin{aligned}&(\omega^2(1+\chi_0) - k^2)E_0^{(s)} + \omega^2\chi_{-g}C^{(s)}E_1^{(s)}\\ &\quad = 8\pi^2\mathrm{i}e\omega\theta VP^{(s)}\delta(\omega - \boldsymbol{kV}),\\ &\omega^2\chi_g C^{(s)}E_0^{(s)} + (\omega^2(1+\chi_0) - (\boldsymbol{k}+\boldsymbol{g})^2)E_1^{(s)} = 0,\end{aligned} \tag{2}$$

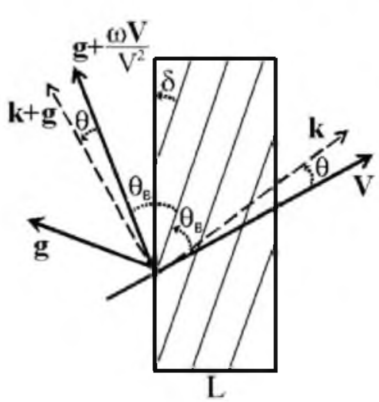

Fig. 1. The geometry of the radiation process on the relativistic electron–monocrystal plate interaction. θ_{B} – Bragg's angle; $\boldsymbol{k}$ – wave vector of the incident pseudo photon; $\boldsymbol{g}$ – reciprocal lattice vector; $\boldsymbol{V}$ – electron velocity vector; θ – angle of photon observation.

where χ_g, χ_{-g}, χ_0 are the factors in expansion of the dielectric susceptibility in the Fourier series by reciprocal lattice vectors $\boldsymbol{g}$

$$\chi(\omega,\mathbf{r}) = \sum_g \chi_g(\omega)\mathrm{e}^{\mathrm{i}gr} = \sum_g(\chi_g'(\omega) + \mathrm{i}\chi_g''(\omega))\mathrm{e}^{\mathrm{i}gr}. \tag{3}$$

We will consider a crystal with the following symmetry: $(\chi_g = \chi_{-g})$.

The values $C^{(s)}$ and $P^{(s)}$ are defined as

$$\begin{aligned}&C^{(s)} = \boldsymbol{e}_0^{(s)}\boldsymbol{e}_1^{(s)}, \quad C^{(1)} = 1, \quad C^{(2)} = \cos(2\theta_{\mathrm{B}}),\\ &P^{(s)} = \boldsymbol{e}_0^{(s)}(\boldsymbol{\rho}/\rho), \quad P^{(1)} = \sin\varphi, \quad P^{(2)} = \cos\varphi,\end{aligned} \tag{4}$$

where $\boldsymbol{\rho} = \boldsymbol{k} - \omega\boldsymbol{V}/V^2$ is the virtual photon momentum vector component perpendicular to the particle velocity vector $\boldsymbol{V}$ ($\rho = \omega\theta/V$, where θ ($\theta \ll 1$) is the angle between $\boldsymbol{k}$ and $\boldsymbol{V}$), θ_{B} is the angle between the electron velocity and a set of atomic planes in the crystal (Bragg angle), φ is the azimuth angle, counted off from the plane formed by vectors $\boldsymbol{V}$ and $\boldsymbol{g}$, the value of reciprocal lattice vector is shown by expression $g = 2\omega_{\mathrm{B}}\sin\theta_{\mathrm{B}}/V$, ω_{B} – Bragg's frequency. Under $s = 1$ the system (2) describes the fields of σ-polarization and under $s = 2$ the fields of π-polarization.

The solution of the system (2) for the coulomb field of relativistic electron in vacuum is like

$$E_0^{(s)\mathrm{vac}} = \frac{8\pi^2\mathrm{i}eV}{\omega}\frac{\theta P^{(s)}}{-\gamma^{-2} - \theta^2}, \tag{5a}$$

and for the coulomb field in crystal (without the PXR contribution) it is like

$$E_0^{(s)\mathrm{cr}} = \frac{8\pi^2\mathrm{i}eV}{\omega}\frac{\theta P^{(s)}}{\chi_0 - \gamma^{-2} - \theta^2}, \tag{5b}$$

where $\gamma = (1 - V^2)^{-1/2}$ is the particle Lorentz factor.

The field of the radiation generated on the entrance surface of the crystal is

$$E_0^{(s)\mathrm{vac-cr}} = \frac{8\pi^2\mathrm{i}eV}{\omega}\theta P^{(s)}\left(\frac{1}{-\gamma^{-2} - \theta^2} - \frac{1}{\chi_0 - \gamma^{-2} - \theta^2}\right). \tag{6a}$$

The one on the exit surface of the crystal is

$$E_0^{(s)\mathrm{cr-vac}} = \frac{8\pi^2\mathrm{i}eV}{\omega}\theta P^{(s)}\left(\frac{1}{\chi_0 - \gamma^{-2} - \theta^2} - \frac{1}{-\gamma^{-2} - \theta^2}\right). \tag{6b}$$

From this it follows, that the electric field of the TR wave radiated ahead of the crystal will be expressed as

$$E_{\mathrm{Rad}}^{(s)\mathrm{TR}} = E_0^{(s)\mathrm{vac-cr}}U^{(s)} + E_0^{(s)\mathrm{cr-vac}}, \tag{7a}$$

$$\begin{aligned}E_{\mathrm{Rad}}^{(s)\mathrm{TR}} &= \frac{8\pi^2\mathrm{i}eV}{\omega}\theta P^{(s)}\\ &\quad\times\left(\frac{1}{\gamma^{-2} + \theta^2} - \frac{1}{\gamma^{-2} + \theta^2 - \chi_0}\right)\left(1 - U^{(s)}\right)\end{aligned} \tag{7b}$$

The amplitude transmission coefficient $U^{(s)}$ defines the field amplitude of TR, radiated on the entrance surface of the crystal and left the crystal through its exit surface. The coefficient $U^{(s)}$ in Bragg's geometry taking into consideration such a view [7] as

$$U^{(s)} = \left(-2\sqrt{z^2+q^{(s)}}\right) \Big/ \left(\left(-z-\sqrt{z^2+q^{(s)}}\right)\right.$$
$$\times e^{\frac{i\omega L}{2\cos(\psi_0)}\left(\chi_0-\theta^2-\gamma^{-2}-z+\sqrt{z^2+q^{(s)}}\right)}$$
$$\left. -\left(-z+\sqrt{z^2+q^{(s)}}\right)e^{\frac{i\omega L}{2\cos(\psi_0)}\left(\chi_0-\theta^2-\gamma^{-2}-z-\sqrt{z^2+q^{(s)}}\right)}\right), \quad (8)$$

where $q^{(s)} = -C^{(s)^2}\chi_g^2/\varepsilon$, $z = -(\alpha-(1+\varepsilon)(\chi_0-\theta^2-\gamma^{-2}))/2\varepsilon$, $\varepsilon = \sin(\theta_B-\delta)/\sin(\theta_B+\delta)$, $\psi_0 = \pi/2-(\theta_B+\delta)$ is the angle between the incident wave vector and the external normal to the target, δ is the angle between the entrance surface of the target and the crystal plane, $\alpha = \frac{1}{\omega^2}((\boldsymbol{k}+\boldsymbol{g})^2-k^2) = \frac{4\sin^2\theta_B}{V^2}\left(\frac{\omega_B(1+\theta\cos\varphi\cot\theta_B)}{\omega}-1\right)$.

The TR spectral–angular distribution can be expressed as [8]

$$\frac{d^2W^{TR}}{d\omega\, d\Omega} = \omega^2(2\pi)^{-6}\sum_{s=1}^{2}\left|E_{Rad}^{(s)^{TR}}\right|^2. \quad (9)$$

By substituting (8) with (7b) and then (7b) with (9), we will draw the expression for the spectral–angular distribution of TR in the crystal:

$$\omega\frac{d^2N^{TR}}{d\omega\, d\Omega} = \frac{e^2}{\pi^2}\theta^2\sum_{s=1}^{2}P^{(s)^2}\left(\frac{1}{\gamma^{-2}+\theta^2}-\frac{1}{\gamma^{-2}+\theta^2-\chi_0'}\right)^2$$
$$\times\left|1+(2K^{(s)})\Big/\left(\left(\left(\zeta^{(s)}-K^{(s)}\right)e^{\frac{i\omega L|\chi_g'|C^{(s)}}{2\cos(\psi_0)}\left(\frac{1}{\varepsilon}\left(\zeta^{(s)}+K^{(s)}\right)-\beta^{(s)}+i\rho^{(s)}\right)}\right.\right.\right.$$
$$\left.\left.\left.-\left(\zeta^{(s)}+K^{(s)}\right)e^{\frac{i\omega L|\chi_g'|C^{(s)}}{2\cos(\psi_0)}\left(\frac{1}{\varepsilon}(\zeta^{(s)}-K^{(s)})-\beta^{(s)}+i\rho^{(s)}\right)}\right)\right)\right|^2, \quad (10)$$

where

$$K^{(s)} = \left(\xi^{(s)^2}-\varepsilon-\frac{\chi_0''}{|\chi_g'|C^{(s)}}\left(1+\varepsilon+2\frac{\chi_g''C^{(s)}}{\chi_0''}\frac{|\chi_g'|}{\chi_g'}\varepsilon\right)\xi^{(s)}i\right.$$
$$\left.-\frac{\chi_0''^2}{\chi_g'^2C^{(s)^2}}\left(\frac{(1+\varepsilon)^2}{4}-\frac{\chi_g''^2C^{(s)^2}}{\chi_0''^2}\right)\right)^{1/2},$$
$$\xi^{(s)} = \eta^{(s)}+\frac{\beta^{(s)}(1+\varepsilon)}{2}, \quad \zeta^{(s)} = \eta^{(s)}+\frac{(\beta^{(s)}-i\rho^{(s)})(1+\varepsilon)}{2},$$
$$\rho^{(s)} = \frac{\chi_0''}{|\chi_g'|C^{(s)}}, \quad \beta^{(s)} = \frac{1}{|\chi_g'|C^{(s)}}(\theta^2+\gamma^{-2}-\chi_0'),$$
$$\eta^{(s)} = \frac{\alpha}{2|\chi_g'|C^{(s)}} = \frac{2\sin^2\theta_B}{V^2|\chi_g'|C^{(s)}}\left(\frac{\omega_B(1+\theta\cos\varphi\cot\theta_B)}{\omega}-1\right). \quad (11)$$

Since in X-ray frequency region the condition $2\sin^2\theta_B/V^2|\chi_g'|C^{(s)} \gg 1$ is held true, the quantity $\eta^{(s)}(\omega)$ depends strongly on frequency, therefore it is very convenient as a spectral variable for analysis of TR spectrum.

The expression (10) can be used in the investigation of dynamic effects manifestation in transition radiation.

3. The dependence of spectral–angular distribution of the transition radiation on radiator orientation

Parameter $\varepsilon = \sin(\theta_B-\delta)/\sin(\theta_B+\delta)$ defines the orientation of the crystal plate surfaces. Under a fixed value of θ_B on the decrease of the electron incidence angle $\theta_B+\delta$ the parameter δ becomes negative and its absolute value increases, which leads to the increasing of parameter ε. On the opposite, on the increase of angle $\theta_B+\delta$ parameter ε decreases (in the extreme case $\delta\to\theta_B$). When $\varepsilon>0$ the crystal is oriented according to Bragg's geometry and the part of TR formed on the entrance surface of the crystal and then diffracted on a system of parallel atomic planes in the crystal leaves the crystal through its entrance surface.

Let us consider the influence of crystal orientation on the spectral–angular properties of TR in the asymptotic case of thin crystal, when the absorption of the radiation is negligible ($\chi_0''=0$). In this case the expression (10) becomes

$$\omega\frac{d^2N^{TR}}{d\omega\, d\Omega} = \frac{e^2}{\pi^2}\theta^2\sum_{s=1}^{2}P^{(s)^2}\left(\frac{1}{\gamma^{-2}+\theta^2}-\frac{1}{\gamma^{-2}+\theta^2-\chi_0'}\right)^2T^{(s)},$$

where

$$T^{(s)} = \left[1+\frac{\xi^{(s)^2}-\varepsilon}{\xi^{(s)^2}-\varepsilon+\varepsilon\sin^2\left(b^{(s)}\frac{\sqrt{\xi^{(s)^2}-\varepsilon}}{\varepsilon}\right)}\right.$$
$$\times\left\langle 1-2\left(\cos\left(b^{(s)}\frac{\sqrt{\xi^{(s)^2}-\varepsilon}}{\varepsilon}\right)\right.\right.$$
$$\times\cos\left(b^{(s)}\left(\frac{\xi^{(s)}}{\varepsilon}-\beta^{(s)}\right)\right)+\frac{\xi^{(s)}}{(\xi^{(s)^2}-\varepsilon)^{1/2}}$$
$$\left.\left.\left.\times\sin\left(b^{(s)}\frac{\sqrt{\xi^{(s)^2}-\varepsilon}}{\varepsilon}\right)\sin\left(b^{(s)}\left(\frac{\xi^{(s)}}{\varepsilon}-\beta^{(s)}\right)\right)\right)\right\rangle\right], \quad (12)$$

$b^{(s)} = \omega L|\chi_g'|C^{(s)}/2\sin(\theta_B+\delta)$ is parameter dependent on crystal thickness; $T^{(s)}$ is the factor describing the interference of the radiations formed on both of the crystal plate boundaries.

It is necessary to note, that in the case, when the surfaces of the plate are parallel to the crystal plane ($\delta=0$ or $\varepsilon=1$) expression (12) transforms to the expression for TR in the dynamic approximation, obtained in [9]. Expression (12) is valid for all possible values of quantity $\eta^{(s)}$ and significantly differs from the formula for TR on an amorphous plate of the same thickness L. Such a difference is stipulated by the diffraction of the radiation on atomic planes and represents a dynamic effect. This difference is significant only in the vicinity of Bragg's frequency $|\eta^{(s)}+\beta^{(s)}(1+\varepsilon)/2|\leqslant\varepsilon^{1/2}$. Out of Bragg's frequency vicinity the condition $|\eta^{(s)}+\beta^{(s)}(1+\varepsilon)/2|\gg\varepsilon^{1/2}$ is valid and expression (12) takes the view of the well known formula for TR formed on an amorphous dielectric plate:

$$\omega\frac{\mathrm{d}^2N^{\mathrm{TR}}}{\mathrm{d}\omega\,\mathrm{d}\Omega}=2\frac{e^2}{\pi^2}\theta^2\sum_{s=1}^{2}P^{(s)^2}\left(\frac{1}{\gamma^{-2}+\theta^2}-\frac{1}{\gamma^{-2}+\theta^2-\chi_0'}\right)^2 \times\left(1-\cos\left(\frac{\omega L}{2\sin(\theta_{\mathrm{B}}+\delta)}(\theta^2+\gamma^{-2}-\chi_0')\right)\right). \tag{13}$$

As we are interested in the dynamic effects, let us consider the frequencies close to Bragg's frequency ω_{B}. From expression (13), in particular, it follows, that the destructive interference of the TR waves, emitted on the entrance and exit surfaces of the crystal plate, leads to a full suppression of radiation in the frequency range close to Bragg's frequency under condition [10]:

$$\frac{\omega L}{2\sin(\theta_{\mathrm{B}}+\delta)}(\theta^2+\gamma^{-2}-\chi_0')=2\pi n, \tag{14}$$

where n is a natural number. In such conditions in the vicinity of Bragg's frequency the radiation, leaving the crystal in a forward direction, will represent a peak without any ordinary TR background in immediate proximity to it. This peak is formed by the uncompensated part of TR from the exit surface of the monocrystal plate. The realization of the destructive interference condition (14) for different values of parameter ε can be accomplished, for example, by the choice of the corresponding values of plate thickness L:

$$L=\frac{4\cdot\pi\cdot n\cdot\sin(\theta_{\mathrm{B}}+\delta)}{\omega\cdot(\theta^2+\gamma^{-2}-\chi_0')} =\frac{4\cdot\pi\cdot n\cdot\sin\left(\theta_{\mathrm{B}}+\arctan\left[\frac{1-\varepsilon}{1+\varepsilon}\tan\theta_{\mathrm{B}}\right]\right)}{\omega\cdot(\theta^2+\gamma^{-2}-\chi_0')}. \tag{15}$$

In Fig. 2 the curves, describing the TR spectrum of σ-polarization, calculated under the anti-resonance condition (14), are presented. The curves are calculated by formula (12) for different values of parameter ε and the fixed value of parameter $\beta^{(s)}$. In the frequency region of anomalous dispersion of the radiation, emitted on the entrance crystal surface (the region of total internal reflection) in a forward direction the peak of the radiation, consisting practically of the radiation emitted on the exit surface only, will be observed. One can define the anomalous dispersion region from expression (11) as

$$-\varepsilon^{1/2}-\beta^{(s)}(1+\varepsilon)/2<\eta^{(s)}<\varepsilon^{1/2}-\beta^{(s)}(1+\varepsilon)/2 \tag{16}$$

and the width of this region can be defined as $2\varepsilon^{1/2}$.

It is seen from the figure that under the increasing of ε the spectral distribution shifts to smaller values of parameter $\eta^{(s)}$, the total internal reflection region widens and the interference peak decreases and under large values of ε practically vanishes. Under the decrease of parameter ε the total reflection region converges and the interference peak increases, that leads to the rise and growth of the interference peak and to the narrowing of its spectral width.

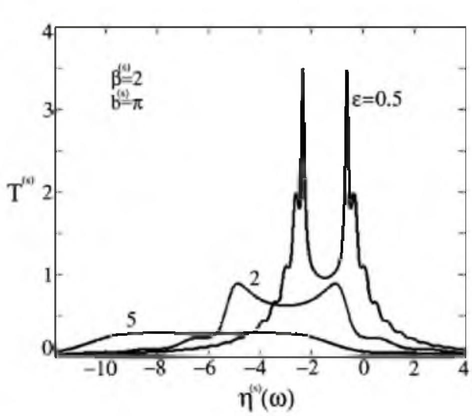

Fig. 2. The influence of crystal plate entrance surface orientation (parameter ε) on the spectral distribution $T(\eta)$ of the relativistic electron TR in a forward direction (on formula (12)).

4. Spectral–angular distribution of diffracted transition radiation

In the preceding section the TR of the relativistic electron crossing the crystal plate was considered in a forward direction. The influence of a diffracted system of parallel atomic planes on the radiation in the crystal was shown. In this chapter we will consider the diffracted transition radiation (DTR), propagating through the entrance surface of the plate in a backward direction. Let us handle the influence of plate surfaces orientation on the spectral–angular properties of the DTR.

The radiation field, emitted by the relativistic electron on the entrance surface (6a) after the diffraction on the system of parallel atom planes in the crystal, is

$$E_{\mathrm{Rad}}^{(s)\mathrm{DTR}}=E_0^{(s)\mathrm{vac-cr}}Q^{(s)} =\frac{8\pi^2\mathrm{i}eV}{\omega}\theta P^{(s)}\left(\frac{1}{-\gamma^{-2}-\theta^2}-\frac{1}{\chi_0-\gamma^{-2}-\theta^2}\right)Q^{(s)}, \tag{17}$$

where $Q^{(s)}$ is the amplitude coefficient of the radiation field reflection of the crystal, given by [7]:

$$Q^{(s)}=\frac{C^{(s)}\chi_g}{\varepsilon}\left(\mathrm{e}^{-\frac{\mathrm{i}\omega L}{2\cos(\psi_0)}\sqrt{z^2+q^{(s)}}}-\mathrm{e}^{\frac{\mathrm{i}\omega L}{2\cos(\psi_0)}\sqrt{z^2+q^{(s)}}}\right)\Bigg/ \left(\left(-z-\sqrt{z^2+q^{(s)}}\right)\mathrm{e}^{\frac{\mathrm{i}\omega L}{2\cos(\psi_0)}\sqrt{z^2+q^{(s)}}} -\left(-z+\sqrt{z^2+q^{(s)}}\right)\mathrm{e}^{-\frac{\mathrm{i}\omega L}{2\cos(\psi_0)}\sqrt{z^2+q^{(s)}}}\right). \tag{18}$$

In the case of a thin crystal, when absorption is neglected, this coefficient can be given by

$$Q^{(s)} = \left(e^{-ib^{(s)}\frac{\sqrt{\xi^{(s)^2}-\varepsilon}}{\varepsilon}} - e^{ib^{(s)}\frac{\sqrt{\xi^{(s)^2}-\varepsilon}}{\varepsilon}}\right) \Big/ \left(\left(\xi^{(s)} - \sqrt{\xi^{(s)^2}-\varepsilon}\right) \times \exp\left(ib^{(s)}\frac{\sqrt{\xi^{(s)^2}-\varepsilon}}{\varepsilon}\right) - \left(\xi^{(s)} + \sqrt{\xi^{(s)^2}-\varepsilon}\right) \times \exp\left(-ib^{(s)}\frac{\sqrt{\xi^{(s)^2}-\varepsilon}}{\varepsilon}\right)\right). \quad (19)$$

To define the spectral–angular density of TR let's substitute (17) to the formula

$$\frac{d^2 W^{\mathrm{DTR}}}{d\omega\, d\Omega} = \omega^2 (2\pi)^{-6} \sum_{s=1}^{2} \left| E_{\mathrm{Rad}}^{(s)\mathrm{DTR}} \right|^2, \quad (20)$$

then we will draw the next

$$\omega \frac{d^2 N^{\mathrm{DTR}}}{d\omega\, d\Omega} = \frac{e^2}{\pi^2}\theta^2 \sum_{s=1}^{2} P^{(s)^2} \left(\frac{1}{\gamma^{-2}+\theta^2} - \frac{1}{\gamma^{-2}+\theta^2-\chi_0'}\right)^2 \left|Q^{(s)}\right|^2. \quad (21)$$

It turns out, that the square module of coefficient (19) can be presented as well-behaved function in the following way:

$$\left|Q^{(s)}\right|^2 = \frac{\sin^2\left(b^{(s)}\frac{\sqrt{\xi^{(s)^2}-\varepsilon}}{\varepsilon}\right)}{\xi^{(s)^2} - \varepsilon + \varepsilon \sin^2\left(b^{(s)}\frac{\sqrt{\xi^{(s)^2}-\varepsilon}}{\varepsilon}\right)}, \quad (22)$$

$\xi^{(s)}$ is defined in (11).

In Fig. 3 the curves describing DTR by formula (22) are presented for different values of parameter ε characterizing the crystal plate orientation. It is seen, that by increasing parameter ε the intensity of DTR decreases.

By substituting (22) for (21) we can obtain the general expression for the spectral–angular distribution of DTR

$$\omega \frac{d^2 N^{\mathrm{DTR}}}{d\omega\, d\Omega} = \frac{e^2}{\pi^2}\theta^2 \sum_{s=1}^{2} P^{(s)^2} \left(\frac{1}{\gamma^{-2}+\theta^2} - \frac{1}{\gamma^{-2}+\theta^2-\chi_0'}\right)^2 \times \frac{\sin^2\left(b^{(s)}\frac{\sqrt{\xi^{(s)^2}-\varepsilon}}{\varepsilon}\right)}{\xi^{(s)^2} - \varepsilon + \varepsilon \sin^2\left(b^{(s)}\frac{\sqrt{\xi^{(s)^2}-\varepsilon}}{\varepsilon}\right)}. \quad (23)$$

It should be noted, that in the case, when the entrance surface of the crystal is parallel to a system of crystal planes, expression (23) turns into the expression for DTR, obtained in [11].

For studying the dependence of the DTR angle density on parameter ε, let us integrate the expression (23) over the frequency function $\xi^{(s)}(\omega)$

$$\frac{dN^{\mathrm{DTR}}}{d\Omega} = \int \frac{d^2 N^{\mathrm{DTR}}}{d\omega\, d\Omega} d\omega = \int \frac{d^2 N^{\mathrm{DTR}}}{d\omega\, d\Omega} \frac{\omega \left|\chi_g'\right| C^{(s)}}{2\sin^2\theta_B} d\xi. \quad (24)$$

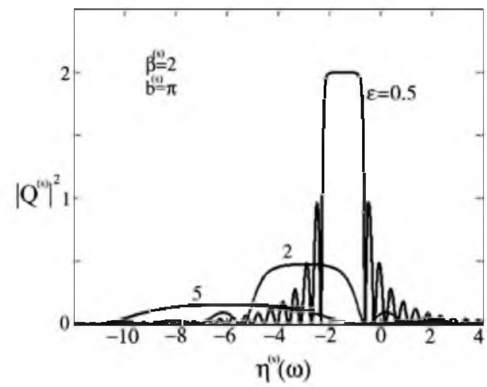

Fig. 3. The influence of crystal plate surface orientation on the spectral distribution of DTR (on formula (22)).

In order to plot the curves of the radiation angle density distribution (in dependence on angle θ) let us rewrite the expression (24) for σ-polarization in the following way:

$$\frac{dN_{\sigma}^{\mathrm{DTR}}}{d\Omega} = \frac{e^2}{\pi^2}\frac{|\chi_g'|C^{(1)}}{2\sin^2\theta_B}\frac{\omega_0^4}{\omega_B^4}F,$$

where

$$F = \frac{\theta_\perp^2}{(\gamma^{-2}+\theta_\perp^2)^2\left(\gamma^{-2}+\theta_\perp^2+\frac{\omega_0^2}{\omega_B^2}\right)^2} \times \int_{-\infty}^{+\infty} \frac{\sin^2\left(b^{(1)}\frac{\sqrt{\xi^{(1)^2}-\varepsilon}}{\varepsilon}\right)}{\xi^{(1)^2} - \varepsilon + \varepsilon \sin^2\left(b^{(1)}\frac{\sqrt{\xi^{(1)^2}-\varepsilon}}{\varepsilon}\right)} d\xi, \quad (25)$$

$\theta_\perp = \theta \sin\varphi$.

The curves, plotted for different values of parameter ε, are presented in Fig. 4. Here one can see that the angle distribution of the DTR density is strongly dependent on ε. It is important, with relation to the problem of creating X-radiation quasimonochromatic sources on the base of relativistic electron-single crystal interaction.

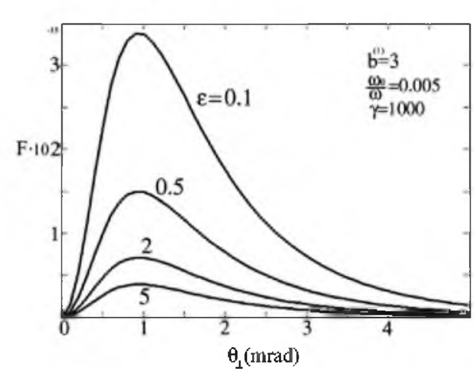

Fig. 4. The angular distribution of the DTR density $F(\theta)$ integrated on photon energy ω calculated for different values of parameter ε (calculated on formula (25)).

5. Conclusions

Thus in this work the detailed theoretical analysis of TR and DTR of the relativistic electron in a monocrystal plate is carried out in Bragg's geometry. Using the dynamic theory of diffraction the expressions for spectral–angular distributions of TR and DTR are deduced in a form, that allows to study the dependence of spectral–angular characteristics on the relative orientation of the crystal lattice and the entrance surface of the plate as a single crystal radiator. The studies showed that the spectral–angular properties of TR and DTR vary significantly with this orientation. This effect is explained by the modification of the frequency total reflection range under a changing surface orientation. It can be used both for the intensity enhancement of the X-radiation source on the base of the RTR mechanism and for the suppression of the transition radiation as a background in the cases when TR hinders the experimental studying of other radiation mechanisms of electron–crystal interaction.

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